

Accuracies of contrasts
between estimated breeding values
of selection candidates
from national cattle evaluations
using pedigree or single-step genomic methodologies

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EBV contrasts



- Evaluations produce individual EBVs for each trait (and typically a corresponding accuracy or reliability).
- ▶ Easy to find the "best" individual....
 - ▶ Rank ordering e.g. sort by highest EBV (or index value).
 - EBV_{SireA} with accuracy/ R^2 .
 - 2. EBV_{SireB} with accuracy/ R^2 .
- But how much better is it?
 - Contrasting EBVs of two (or more) animals.
 - ► EBV_{SireA} EBV_{SireB} with accuracy of?





- ▶ 2,118,874 animals in the pedigree.
- ▶ 1,416,006 birthweight observations.
- ▶ 38,175 genotypes using the MSRP subset (Saatchi & Garrick 2014).
- ▶ MMEs solved (PCG) and sampled (MCMC) using BOLT software.
- ▶ 80,000 MCMC samples of plausible values of every effect stored.
- ▶ EBVs are posterior means and PEVs are posterior variances of the chain of samples.

Pedigree BLUP



$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{Z}\mathbf{u} + \mathbf{Z}_{\mathbf{m}}\mathbf{m} + \mathbf{Z}_{\mathbf{p}}\mathbf{p} + \mathbf{e}$$

Fixed effects (b), direct effects (u), maternal genetic (m), maternal permanent environment (p), and residual (e) effects

Super Hybrid Model



$$\begin{bmatrix} \mathbf{y_n} \\ \mathbf{y_g} \end{bmatrix} = \begin{bmatrix} \mathbf{X_n} \\ \mathbf{X_g} \end{bmatrix} \mathbf{b} + \begin{bmatrix} \mathbf{Z_n} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z_g} \end{bmatrix} \begin{bmatrix} \mathbf{M_n} \alpha + \varepsilon \\ \mathbf{M_g} \alpha \end{bmatrix} + \mathbf{Zu} + \mathbf{Z_m} \mathbf{m} + \mathbf{Z_p} \mathbf{p} + \mathbf{e}$$

- Super Hybrid Model (Fernando et al. 2016) for genotyped (g) and non-genotyped (n) animals.
- Includes marker effects (α) .
- For non-genotyped animals, uses imputed markers (\mathbf{M}_n) and fits an imputation error term (ε).





- Calculated from diagonal and off-diagonal elements of the prediction error variance matrix.
 - ▶ PEV matrix is the inverse of the LHS of MME.
 - ▶ Inverse is computationally prohibitive, especially in single-step.
- ▶ Can approximate diagonal elements of inverse but PEV of contrasts rely on arbitrary off-diagonal elements
- Avoid approximation with MCMC (e.g. BOLT software).
 - ▶ GPU-accelerated single-site Gibbs sampler.





- One column per animal of interest that contains its chain of plausible EBVs.
- ▶ Make chain of contrast of samples, e.g. EBV_{SireA} EBV_{SireB}

$$\begin{bmatrix} EBV_{1,SireA}EBV_{1,SireB}\cdots EBV_{1,SireZ}\\ \vdots\\ \vdots\\ EBV_{N,SireA}EBV_{N,SireB}\cdots EBV_{N,SireZ} \end{bmatrix} \begin{bmatrix} 1\\ -1\\ 0\\ \vdots\\ 0 \end{bmatrix} \rightarrow \begin{array}{l} \text{Contrast}\\ \text{vector } \textbf{\textit{k}} \\ \text{echain of EBV}_{SireA} - \text{EBV}_{SireB} \\ \vdots\\ 0 \end{bmatrix}$$

- ▶ EBV of the contrast is the mean of the chain of contrasts.
- ▶ PEV of the contrast is the variance of the chain of contrasts.





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▶ Results for "high" accuracy sires

		PBLUP			SHI	M	
	ID	EBV	\mathbb{R}^2	BIF	EBV	\mathbb{R}^2	BIF
High população J	A	0.71	0.97	0.84	0.84	0.97	0.84
High accuracy sires $\stackrel{\leftarrow}{\leftarrow}$	В	3.04	0.96	0.81	3.15	0.97	0.81





▶ Genomic information improves low accuracy.

		PBLUP			SH	M		
	ID	EBV	\mathbb{R}^2	BIF	EBV	\mathbb{R}^2	BIF	
	A	0.71	0.97	0.84	0.84	0.97	0.84	
High accuracy sires	В	3.04	0.96	0.81	3.15	0.97	0.81	
	С	0.10	0.60	0.37	-0.46	0.72	0.47	Г
	D	-0.03	0.59	0.36	0.36	0.72	0.47	
	E	1.17	0.60	0.37	1.47	0.73	0.48	
	F	-1.86	0.61	0.37	-1.81	73	0.48	Г
	G	3.48	0.62	0.38	Benefit o	Benefit of		
	Н	-0.03	0.59	0.36	genotyping y	Oling	0.47	
2016 horn males	I	-2.91	0.59	0.36	animals		0.47	Г
2016 born males	J	0.95	0.59	0.36			0.47	
	K	-1.73	0.59	0.36	-1.00	<i>J.</i> 72	0.47	
	L	1.17	0.60	0.37	1.47	0.73	0.48	
	M	-0.72	0.62	0.38	-1.70	0.73	0.48	
	N	0.95	0.59	0.36	1.55	0.72	0.47	
	O	0.22	0.62	0.38	0.29	0.73	0.49	
L. C.	P	-1.21	0.58	0.35	-0.88	0.70	0.45	
					L			4





		PBLUP			SHM		
Contrast	var(k'u)	PEV	\mathbb{R}^2	BIF	PEV	R^2	BIF
						-	
B-A	63.12	1.92	0.97	0.83	1.82	0.97	0.83

High accuracy sires have high accuracy contrast (in this case)



Results – Contrasts same herd

		PBLUP			SHM		
Contrast	var (k'u)	PEV	\mathbb{R}^2	BIF	PEV	\mathbb{R}^2	BIF
D-C	43.40	24.09	0.44	0.25	16.68	0.62	0.38
С-Е	40.12	23.84	0.41	0.23	16.39	0.59	0.36
G-F	55.72	24.32	0.56	0.34	16.65	0.70	0.45
H-G	59.85	24.53	0.59	0.36	16.58	0.72	0.47

- ▶ Young selection candidates with same sire
- **▶** Young selection candidates with different sires



Results - Contrasts different herd

		PBLUP			SHM		
Contrast	var (k'u)	PEV	\mathbb{R}^2	BIF	PEV	\mathbb{R}^2	BIF
J-I	44.28	24.78	0.44	0.25	16.98	0.62	0.38
L-K	44.51	24.26	0.46	0.26	16.68	0.63	0.39
N-M	56.11	24.55	0.56	0.34	16.87	0.70	0.45
P-O	59.29	25.11	0.58	0.35	17.77	0.70	0.45

- Young selection candidates with same sire
- Young selection candidates with different sires
- ▶ Genomic information improves accuracy of contrasts.
- Contrast between animals with the same sire have lower accuracy than those with different sires regardless of herd since PEV similar but var(k'u) lower.
- ▶ These herds are well-connected due to wide Al use.

Conclusions



- ▶ For comparing animals, it is the contrast (and the accuracy/reliability of contrasts) that matters.
- Accuracy of the individuals EBV's are not an indication of the accuracy of the contrast.
 - ▶ Depends on prediction error co-variances which are influenced by "connectedness".
- MCMC sampling of the MME using BOLT software is a computationally efficient method for national animal evaluations.
 - ▶ Same MCMC principles can be applied to selection indexes.

Questions?



Special thanks to the American Hereford Association for allowing the use of their national single-step evaluation data.







- Individual EBVs reported with a corresponding accuracy
 - ▶ Reliability R²
 - ▶ Accuracy r=sqrt(R²)
 - ▶ BIF accuracy
- "Measure" of the amount of information that went into producing the EBV
 - Quantify the possible variation of the EBV
- Prediction error variance (PEV)
 - ▶ Elements of inverse of LHS of MME
 - Inverse is prohibitive to compute, especially in single-step
 - PEVs commonly approximated





- For a contrast vector **k**, accuracy is computed as:
- $Arr R^2 = I PEV_k / (k'Gk)$ and $BIF = I sqrt(I R^2)$
- Var(k'u)=k'Gk, where G=var(u)= σ_g^2A for single trait where A is the numerator relationship matrix
- For demonstration purposes var(k'u) is taken to be the same in both the PBLUP and SHM (ie k'Gk)

Simplified hypothetical example

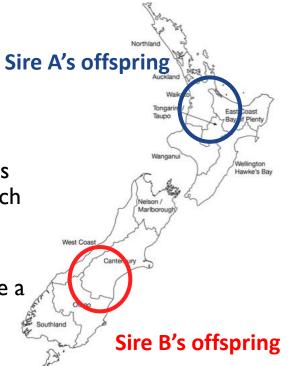


- Sire A and Sire B
- Both sires have many offspring.
- Both sires have high accuracy.

But, offspring are in their own herds and geographically isolated from each other.

A contrast of these sires might have a low or high accuracy.

▶ Even in the absence of GxE



https://geog397.wiki.otago.ac.nz/images/9/92/MAP.jpg

Overview



- Why EBV contrasts are important.
- Single trait birthweight model
 - ▶ Pedigree BLUP genetic evaluation.
 - ▶ Single-step Super Hybrid Model (SHM) genomic evaluation.
- Accuracy of contrasts.
- Results.
 - ▶ Contrast of high accuracy sires.
 - Contrasts of 2016 born males.
- Conclusions.